Cheeger-Gromov L^2 ρ-invariants of 3-manifolds

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L^2 ρ-invariants

- **Topological Definition of L^2 ρ-invariants, $\rho^{(2)}(M,\phi)$**

  M is a closed (4k-1)-manifold.

  $\phi : \pi_1(M) \rightarrow G$ is a homomorphism.

  Suppose there are a 4k-manifold W such that $\partial W = M$ and a group Γ which make the following diagram commute:

$$\begin{array}{ccc} \pi_1(M) = \pi_1(M) & \xrightarrow{\ \phi\ } & G \\ \quad\quad\quad\downarrow{\scriptstyle i_*} & & \vdots \\ \pi_1(W) & \cdots\cdots\cdots > & \Gamma \end{array}$$

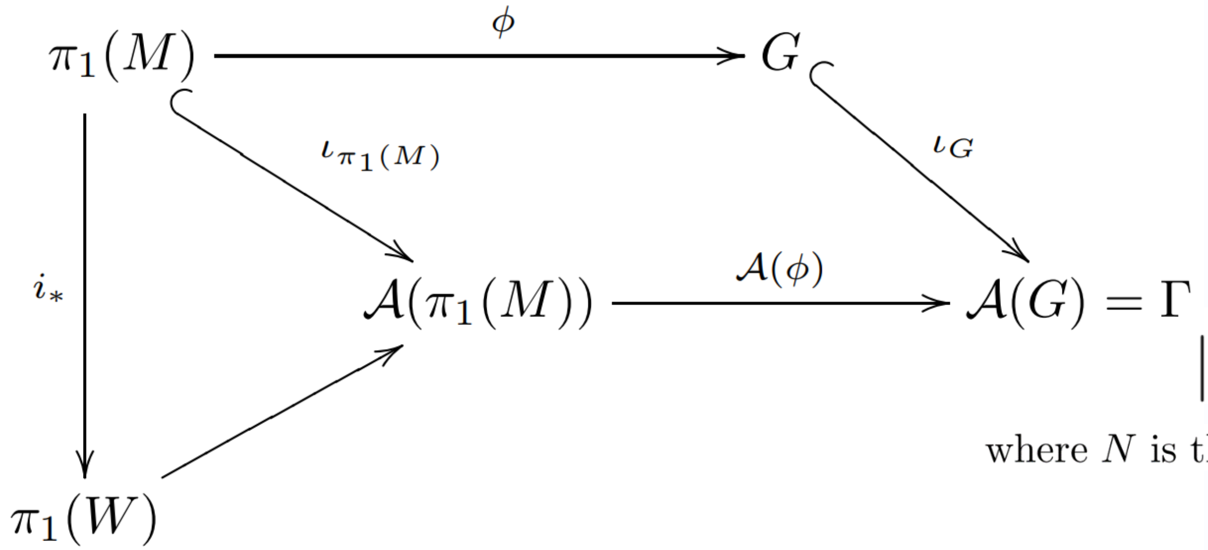
Then, $\rho^{(2)}(M,\phi) := \operatorname{sign}^{(2)}_{\Gamma} W - \operatorname{sign} W$

Topological approach to L^2 ρ-invariants

- **Theorem [Cha 16]**

For a closed 3-manifold M with simplicial complexity n,

$|\rho^{(2)}(M, \phi)| \leq 363090n$ for any homomorphism $\phi : \pi_1(M) \to G$.



$$|\rho^{(2)}(M, \phi)| \leq 2N.$$

where N is the number of the $2k$-cells of W.

- **Theorem [L, work in progress]**

For a spherical 3-manifold M with simplicial complexity n,

$|\rho^{(2)}(M, \phi)| \leq 2340n$ for any homomorphism $\phi : \pi_1(M) \to G$.